Complexity of Bribery and Control for Uniform Premise-Based Quota Rules Under Various Preference Types

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Abstract. Manipulation of judgment aggregation procedures has first been studied by List [14] and Dietrich and List [8], and Endriss et al. [9] were the first to study it from a computational perspective. Baumeister et al. [2,3,6] introduced the concepts of bribery and control in judgment aggregation and studied their algorithmic and complexity-theoretic properties. However, their results are restricted to Hamming-distance-respecting preferences and their results on bribery apply to the premise-based procedure only. We extend these results to more general preference notions, including closeness-respecting and top-respecting preferences that are due to Dietrich and List and have been applied to manipulation in judgment aggregation by Baumeister et al. [4,5]. In addition, our results apply to uniform premise-based quota rules that generalize the premise-based procedure.

Keywords: Bribery · Control · Judgment aggregation · Computational complexity

1 Introduction

Judgment aggregation refers to methods of collective decision making where the judgments of a number of judges are aggregated so as to arrive at a collective judgment set. Endriss et al. [9] were the first to study manipulation in judgment aggregation from a computational point of view. We study the complexity of problems related to bribery and control in judgment aggregation, notions that were introduced and applied to voting problems in computational social choice by Bartholdi et al. [1] (see also the work of Hemaspaandra et al. [13]) for control and by Faliszewski et al. [10,11] for bribery (see also the book chapter by Faliszewski and Rothe [12] for many more references). These notions have been transferred to (a computational study of) judgment aggregation by Baumeister et al. [2,3, 5,6]. However, their results apply to Hamming-distance-respecting preferences only, and in the case of bribery they have only investigated the premise-based procedure. The main contribution of this paper is to extend their study for three types of control (control by adding, by deleting, and by replacing judges)

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and for bribery and microbribery to more general preference notions, including closeness-respecting and top-respecting preferences. We also extend the study of bribery to uniform premise-based quota rules, which generalize the premise-based procedure.

Closeness-respecting and top-respecting preferences have been introduced by Dietrich and List [8] and have been applied to manipulation in judgment aggregation by them and by Baumeister et al. [4,5]. Intuitively, for top-respecting preferences all we know is that the attacker prefers her desired set to any other judgment set, while in closeness-respecting preferences we also know that judgment sets with additional agreements are preferred.

In Sect. 2, we provide the needed notions from judgment aggregation. We study the complexity of control problems in Sect. 3 and that of bribery problems in Sect. 4. In Sect. 5, we summarize our results and propose some open questions for future work.

2 Definitions and Notations

Throughout this paper, we will utilize the judgment aggregation framework due to Endriss et al. [9]. Let $\mathscr{L}_{PS}$ be the set of all propositional formulas that can be built from a set of propositional variables, PS, using the common boolean connectives, i.e., *disjunction* ($\vee$), *conjunction* ($\wedge$), *implication* ($\rightarrow$), and *equivalence* ($\leftrightarrow$) as well as the constants 1 (*true*) and 0 (*false*). We use $\overline{\alpha}$ to refer to the complement of α, that is, $\overline{\alpha} = \neg\alpha$ if α is not negated, and $\overline{\alpha} = \beta$ if $\alpha = \neg\beta$. A set $\Phi \subseteq \mathscr{L}_{PS}$ is said to be *closed under complementation* if $\overline{\alpha} \in \Phi$ for all $\alpha \in \Phi$, and to be *closed under propositional variables* if $PS \subseteq \Phi$. We call a finite nonempty set $\Phi \subseteq \mathscr{L}_{PS}$ without doubly negated formulas that is closed under complementation an *agenda*, and a subset $J \subseteq \Phi$ a *judgment set for Φ*. J is an *individual judgment set* if it is the set of propositions accepted by some judge. Furthermore, J is called *complete* if $\alpha \in J$ or $\overline{\alpha} \in J$ for all $\alpha \in \Phi$, and J is said to be *consistent* if there exists an assignment such that all formulas in J are satisfied. Let $\mathscr{J}(\Phi)$ be the set of all complete and consistent judgment sets of an agenda Φ and let $N = \{1, \dots, n\}$ be the set of judges. We call $\mathbf{J} = (J_1, \dots, J_n) \in \mathscr{J}(\Phi)^n$ the *profile* of the judges' individual judgment sets. A resolute[1] *(judgment aggregation) procedure* for an agenda Φ and a set of judges N of size n is a function $F : \mathscr{J}(\Phi)^n \rightarrow 2^{\Phi}$, where 2^{Φ} denotes the power set of Φ. That means that a procedure maps a profile to a *collective judgment set* or *(collective) outcome.*

Let $\|S\|$ be the cardinality of the set S and let $\models$ denote the satisfaction relation. Dietrich and List [7] introduced the class of premise-based quota rules. We will consider only a special case, the uniform premise-based quota rules.

[1] There are also irresolute judgment aggregation procedures (i.e., procedures that may output more than one collective judgment set), such as the distance-based procedures introduced by Pigozzi [17] and Miller and Osherson [15], which we won't consider here, though.

Definition 1 (Uniform Premise-based Quota Rule). *Let the agenda Φ be closed under propositional variables. Subdivide Φ into the two disjoint subsets Φ_p (the set of* premises*) containing exactly all literals, and Φ_c (the set of* conclusions*), both closed under complementation. Furthermore, subdivide Φ_p into two disjoint subsets, Φ_1 and Φ_2, satisfying that $\varphi \in \Phi_1$ if and only if $\overline{\varphi} \in \Phi_2$. Assign to each literal $\varphi \in \Phi_1$ a rational quota q, $0 \leq q < 1$, and to each literal $\overline{\varphi} \in \Phi_2$ the associated quota $q' = 1 - q$. The* uniform premise-based quota rule with quota q *(denoted by $UPQR_q$) is the procedure mapping each profile $\mathbf{J} = (J_1, \dots, J_n)$ of individual judgment sets for Φ to the collective outcome $UPQR_q(\mathbf{J}) = \triangle \cup \{\psi \in \Phi_c \mid \triangle \models \psi\}$, where $\triangle = \{\varphi \in \Phi_1 \mid \|\{i \mid \varphi \in J_i\}\| > nq\} \cup \{\varphi \in \Phi_2 \mid \|\{i \mid \varphi \in J_i\}\| \geq nq'\}$.*

Throughout the paper, we will assume that all literals in Φ_1 are not negated. Since Φ is closed under propositional variables and Φ_p contains exactly all literals, the outcomes of $UPQR_q$ are complete and consistent. The threshold for a literal $\varphi \in \Phi_1$ to be accepted is $\lfloor nq+1 \rfloor$, i.e., φ is contained in the collective outcome if and only if it is contained in at least $\lfloor nq+1 \rfloor$ individual judgment sets, whereas literals $\overline{\varphi} \in \Phi_2$ need at least $\lceil nq' \rceil$ affirmations. It is possible to determine in polynomial time whether a given formula is an element of the collective outcome of a uniform premise-based quota rule. The special case of $UPQR_{1/2}$ for an odd number of judges is also known as the premise-based procedure (PBP).

We will study judgment aggregation problems where some external agent tries to influence a judgment aggregation process in order to obtain a better outcome. In order to compare two outcomes, we will use various notions of preference types induced by an external agent's desired set. These notions have been introduced by Dietrich and List [8] and have later been refined by Baumeister et al. [5]. Formally, this desired set is a subset of a complete and consistent judgment set.

Let Φ be an agenda, $X, Y \in \mathscr{J}(\Phi)$, and let $\succsim$ be a weak order over $\mathscr{J}(\Phi)$, i.e., a transitive and total binary relation over complete and consistent judgment sets. We say that X *is weakly preferred to* Y whenever $X \succsim Y$, and we say that X *is preferred to* Y, denoted by $X \succ Y$, whenever $X \succsim Y$ and $Y \not\succsim X$. Furthermore, we define $X \sim Y$ by $X \succsim Y$ and $Y \succsim X$.

Definition 2. *Let Φ be an agenda, let U be the set of all weak orders over $\mathscr{J}(\Phi)$, and let J be a possibly incomplete judgment set. Define*

1. *the set $U_J \subseteq U$ of unrestricted J-induced (weak) preferences by*

$$U_J = \{\succsim \ \in U \mid \text{for all } X, Y \in \mathscr{J}(\Phi),\ X \sim Y \text{ whenever } X \cap J = Y \cap J\};$$

2. *the set $TR_J \subseteq U_J$ of top-respecting J-induced (weak) preferences by*

$$TR_J = \left\{ \succsim \ \in U_J \middle| \begin{array}{l} \text{for all } X, Y \in \mathscr{J}(\Phi),\ X \succ Y \\ \text{whenever } X \cap J = J \text{ and } Y \cap J \neq J \end{array} \right\};$$

3. *the set $CR_J \subseteq U_J$ of closeness-respecting J-induced (weak) preferences by*

$$CR_J = \{\succsim \ \in U_J \mid \text{for all } X, Y \in \mathscr{J}(\Phi), \text{ if } Y \cap J \subseteq X \cap J \text{ then } X \succsim Y\}.$$

Definition 3. *Let Φ be an agenda, let X and Y be complete and consistent judgment sets for Φ, let J be an external agent's desired set, and let $T_J \in \{U_J, TR_J, CR_J\}$ be a type of J-induced (weak) preferences. We say that*

1. the external agent *necessarily/possibly weakly prefers X to Y for type T_J if* $X \succsim Y$ for all/some $\succsim\ \in T_J$.
2. the external agent *necessarily/possibly prefers X to Y for type T_J if $X \succ Y$* for all/some $\succsim\ \in T_J$.

Let J be the desired set of the external agent. In the case of closeness-respecting preferences, the external agent necessarily prefers a new outcome Y to the actual outcome X if and only if she achieves a new agreement with J while preserving the existing agreements. On the other hand, she possibly prefers Y to X if and only if she achieves a new agreement with J regardless of new differences.

Example 4. Let $\Phi = \{a,\ b,\ c,\ a \wedge b,\ \neg a \vee c,\ \neg a,\ \neg b,\ \neg c,\ \neg(a \wedge b),\ \neg(\neg a \vee c)\}$ be an agenda and let $\mathbf{J} = (J_1, J_2, J_3)$ be a profile. Table 1 shows the individual judgment sets of the three judges as well as the collective outcome $UPQR_{1/2}(\mathbf{J})$ and the external agent's incomplete desired set J. Here a 1 indicates that the formula is contained in the judgment set, whereas a 0 means that the formula's complement is in the set. Assume the external agent changes the profile to some (not further specified) profile $\mathbf{J}'$ with $UPQR_{1/2}(\mathbf{J}') = \{\neg a, \neg b, c, \neg(a \wedge b), \neg a \vee c\}$ and consider closeness-respecting preferences. Since it holds that $\{\neg(a \wedge b), \neg a \vee c\} = J \cap UPQR_{1/2}(\mathbf{J}') \supset J \cap UPQR_{1/2}(\mathbf{J}) = \{\neg a \vee c\}$, the external agent necessarily prefers $UPQR_{1/2}(\mathbf{J}')$ to $UPQR_{1/2}(\mathbf{J})$.

Table 1. Example for closeness-respecting preferences

Judgment set	a	b	c	$a \wedge b$	$\neg a \vee c$
J_1	1	1	0	1	0
J_2	1	0	1	0	1
J_3	0	1	1	0	1
$UPQR_{1/2}$	1	1	1	1	1
J			0	0	1

We assume that the reader is familiar with the complexity classes P and NP as well as with the concept of polynomial-time many-one reducibility (denoted by $\leq_{m}^{p}$; see, for example, the textbooks by Papadimitriou [16] and Rothe [18]).

We will use the following three NP-complete decision problems in our reductions. Given a propositional formula φ in conjunctive normal form (CNF) so that neither setting all variables to true nor setting all variables to false will satisfy the formula, the problem Restricted-SAT asks whether there is a satisfying assignment for φ. The problem Dominating-Set asks, given a graph

$G = (V, E)$ and a positive integer k, if G has a dominating set of size at most k, i.e., a subset $V' \subseteq V$ where $\|V'\| \le k$ such that every vertex $v \in V$ belongs to the closed neighborhood of some $v' \in V'$. Finally, given a set X and a collection C containing 3-element subsets of X, the problem Exact-Cover-by-3-Sets (X3C) asks if there is an exact cover for X, i.e., a subcollection $C' \subseteq C$ such that each element of X is a member of exactly one element of C'.

3 Control

In this section, we study the complexity of control problems related to the types of preferences defined in the previous section. These types of control in judgment aggregation have been introduced by Baumeister et al. [2,3], but their results are restricted to Hamming-distance-respecting preferences only. Hamming-distance-respecting preferences induce a weak order over all complete and consistent judgment sets for a given agenda, by counting the number of positive formulas on which two judgment sets differ.

3.1 Preliminaries

We now formally define the relevant control problems for the uniform premise-based quota rule with quota q and for some given preference type T, starting with (possible and necessary) control by adding and by deleting judges.

$UPQR_q$-T-Possible-Control-by-Adding-Judges	
Given:	An agenda Φ, two profiles $\mathbf{J} \in \mathscr{J}(\Phi)^n$ and $\mathbf{K} \in \mathscr{J}(\Phi)^m$, a desired set J, and a positive integer k.
Question:	Is there a subprofile $\mathbf{K}' \subseteq \mathbf{K}$ of size at most k such that for the new profile $\mathbf{J}' = \mathbf{J} \cup \mathbf{K}'$, it holds that $UPQR_q(\mathbf{J}') \succ UPQR_q(\mathbf{J})$ for some $\succsim \in T_J$?

$UPQR_q$-T-Possible-Control-by-Deleting-Judges	
Given:	An agenda Φ, a profile $\mathbf{J} \in \mathscr{J}(\Phi)^n$, a desired set J, and a positive integer k.
Question:	Is there a subprofile $\mathbf{J}' \subseteq \mathbf{J}$ of size at most k such that $UPQR_q(\mathbf{J} \smallsetminus \mathbf{J}') \succ UPQR_q(\mathbf{J})$ for some $\succsim \in T_J$?

The next control type, control by replacing judges, combines the previous two types. To motivate this control type, Baumeister et al. [2,3] provide real-world examples taken from the regulations on implementing powers in the Council of the European Union or the European Commission.

Concerning the problems $UPQR_q$-T-Necessary-Control-by-$\mathscr{C}$ for any one of these control types $\mathscr{C}$, the respective condition must hold for *all* $\succsim$ in T_J,

$UPQR_q$-T-Possible-Control-by-Replacing-Judges	
Given:	An agenda Φ, two profiles $\mathbf{J} \in \mathscr{J}(\Phi)^n$ and $\mathbf{K} \in \mathscr{J}(\Phi)^m$, a desired set J, and a positive integer k.
Question:	Are there subprofiles $\mathbf{J}' \subseteq \mathbf{J}$ and $\mathbf{K}' \subseteq \mathbf{K}$ of size $\|\mathbf{J}'\| = \|\mathbf{K}'\| \leq k$ such that for the new profile $\mathbf{S} = (\mathbf{J} \smallsetminus \mathbf{J}') \cup \mathbf{K}'$, it holds that $UPQR_q(\mathbf{S}) \succ UPQR_q(\mathbf{J})$ for some $\succsim \in T_J$?

whereas in $UPQR_q$-Exact-Control-by-$\mathscr{C}$ we ask whether the desired set J is contained in the collective outcome after the external agent (called the chair) has exerted control of type $\mathscr{C}$.

A complete desired set J is a special case of an incomplete one. That means that every NP-hardness result for problems with a complete J automatically shows NP-hardness for the problems with incomplete J. It is easy to see that all decision problems in this section are in NP.

Definition 5. *Let Φ be an agenda and let $\mathscr{C}$ be a given control type. A resolute judgment aggregation procedure F is necessarily/possibly immune to control by $\mathscr{C}$ for induced preferences of type $T \in \{U, TR, CR\}$ if for all profiles $\mathbf{J}$ and for each desired set J, the chair necessarily/possibly weakly prefers the outcome $F(\mathbf{J})$ to the outcome $F(\mathbf{J}')$ for type T_J, where $\mathbf{J}'$ denotes the new profile after exerting control of type $\mathscr{C}$.*

3.2 Results for Control

For uniform premise-based quota rules, Baumeister et al. [3] show NP-completeness of exact control by adding and by deleting judges for complete desired sets and for the quota $q = 1/2$, and NP-completeness of exact control by replacing judges for any quota. The proof of the latter result can be modified so as to use a complete desired set.

Our first result gives a link between the exact control problem of a given type and the corresponding possible and necessary control problem with respect to various preference types induced by the chair's desired set.

Proposition 6. *Let $\mathscr{C}$ be a control type and let q, $0 \leq q < 1$, be a rational quota.*

1. *$UPQR_q$-Exact-Control-by-$\mathscr{C}$ $\leq_m^p$ $UPQR_q$-T-Possible-Control-by-$\mathscr{C}$ for each preference type $T \in \{U, TR, CR\}$.*
2. *$UPQR_q$-Exact-Control-by-$\mathscr{C}$ $\leq_m^p$ $UPQR_q$-T-Necessary-Control-by-$\mathscr{C}$ for each preference type $T \in \{TR, CR\}$.*

The simple proof is an adaption of the proof of the corresponding reductions between certain manipulation problems due to Baumeister et al. [5, Thm. 7]. In the construction we use the conjunction of all formulas in the desired set of the Exact-Control-by-$\mathscr{C}$ instance as the single element of the desired set in the corresponding preference-based instance. Since the latter set is incomplete, the case of complete desired sets has to be considered separately.

Assuming a complete desired set inducing top-respecting preferences, the chair necessarily prefers only her desired set to any other possible outcome. Thus, in this case, NP-completeness of *UPQR*$_q$-*TR*-Necessary-Control-by-$\mathscr{C}$ follows from NP-completeness of the exact control problem of type $\mathscr{C}$ for a complete desired set.

We now consider closeness-respecting preferences.

Theorem 7. *UPQR*$_{1/2}$-*CR*-Possible-Control-by-Adding-Judges *and* *UPQR*$_{1/2}$-*CR*-Necessary-Control-by-Adding-Judges *both are* NP-*complete, even for a complete desired set.*

Table 2. Construction for the proof of Theorem 7

Judgment set	α_0	α_1	$\cdots$	α_{3m}	β	$\varphi \vee \beta$
J_1	1	1	$\cdots$	1	0	1
$J_2, \dots, J_m$	0	1	$\cdots$	1	0	0
J_{m+1}	0	0	$\cdots$	0	0	0
$UPQR_{1/2}$	0	1	$\cdots$	1	0	0
J	0	1	$\cdots$	1	1	1

Proof. The proof works by a reduction from X3C and uses a construction similar to the one employed by Baumeister et al. [3]. Let (X, C) be an X3C instance, where $X = \{x_1, \dots, x_{3m}\}$ and $C = \{C_1, \dots, C_n\}$. For the first part of the theorem, let the agenda Φ contain the literals $\alpha_0, \alpha_1, \dots, \alpha_{3m}, \beta$, the formula $\varphi \vee \beta$ with $\varphi = \alpha_0 \wedge \cdots \wedge \alpha_{3m}$, and the corresponding negations. The profile $\mathbf{J} = (J_1, \dots, J_{m+1})$, the collective judgment set $UPQR_{1/2}(\mathbf{J})$, and the desired set J can be seen in Table 2.

Let $\mathbf{K} = (K_1, \dots, K_n)$ be the profile containing the individual judgment sets to be added, where $K_i = \{\neg\beta, \alpha_0, \alpha_j, \neg\alpha_l \mid x_j \in C_i, x_l \notin C_i, 1 \leq j, l \leq 3m\}$. The chair is allowed to add m judgment sets from $\mathbf{K}$.

Since no judge accepts β, the additional agreement of the new outcome with J can only occur for the formula $\varphi \vee \beta$. To add α_0, the chair has to add at least m judges for a total of $2m + 1$ judges. But then every α_i, $1 \leq i \leq 3m$, needs exactly one additional affirmation. Therefore, there is a successful control if and only if there is an exact cover of the given X3C instance. This shows that *UPQR*$_{1/2}$-*CR*-Possible-Control-by-Adding-Judges is NP-complete.

Concerning the proof of the second part, let the agenda Φ' contain only α_0, α_1, …, α_{3m} and the corresponding negations. Let $\mathbf{J}'$ and $\mathbf{K}'$ be the corresponding profiles restricted to Φ' and let $J' = \{\alpha_0, \alpha_1, \dots, \alpha_{3m}\}$ be the chair's desired set. Since the chair has to preserve the initial agreements with J, by a similar argumentation as above, there is a successful control if and only if there is an exact cover for the given X3C instance. Thus $UPQR_{1/2}$-CR-Necessary-Control-by-Adding-Judges is NP-complete. □

Theorem 8. *$UPQR_{1/2}$-CR-*Possible-Control-by-Deleting-Judges *and $UPQR_{1/2}$-CR-*Necessary-Control-by-Deleting-Judges *both are* NP*-complete, even for a complete desired set.*

Proof. We adapt a construction used by Baumeister et al. [3]. Let (X, C) be an X3C instance, where $X = \{x_1, \ldots, x_{3m}\}$ and $C = \{C_1, \ldots, C_n\}$. If there exists an element of X that is not contained in any element of C, we construct an arbitrary no-instance for the respective control problem.

For the first part, let Φ be the agenda containing the literals $\alpha_0, \alpha_1, \ldots, \alpha_{3m}$, β, γ, the formula $\varphi \vee \beta$ where $\varphi = \alpha_0 \wedge \cdots \wedge \alpha_{3m} \wedge \neg\gamma$, and all corresponding negations. Let $\mathbf{T} = \mathbf{T_1} \cup \mathbf{T_2}$ be a profile where $\mathbf{T_1} = (J_1, \ldots, J_{n+m})$ and $\mathbf{T_2} = (L_1, \ldots, L_n)$ for a total of $2n + m$ judges. We denote by d_k the number of sets C_i that contain x_k. For each i, $1 \leq i \leq n + m$, J_i is the union of the set $\{\neg\beta, \alpha_j, \neg\alpha_l \mid m + d_j \geq i, m + d_l < i, 1 \leq j, l \leq 3m\}$ with $\{\alpha_0\}$ if $i \leq n + 1$ (and with $\{\neg\alpha_0\}$ otherwise), and with $\{\gamma\}$ if $i \leq m$ (and with $\{\neg\gamma\}$ otherwise), and with the corresponding conclusion $\{\varphi \vee \beta\}$ (respectively, with $\{\neg(\varphi \vee \beta)\}$). Furthermore, for $1 \leq i \leq n$, define

$$L_i = \{\neg\beta, \gamma, \neg\alpha_0, \alpha_j, \neg\alpha_l, \neg(\varphi \vee \beta) \mid x_j \notin C_i, x_l \in C_i, 1 \leq j, l \leq 3m\}.$$

Since β has no affirmation, γ and every a_k, $1 \leq k \leq 3m$, each have $n + m$ affirmations, and since α_0 has $n + 1$ affirmations, it follows that

$$UPQR_{1/2}(\mathbf{T}) = \{\neg\alpha_0, \alpha_1, \ldots, \alpha_{3m}, \neg\beta, \gamma, \neg(\varphi \vee \beta)\}.$$

Let the chair's desired set be $J = \{\neg\alpha_0, \alpha_1, \ldots, \alpha_{3m}, \beta, \gamma, \varphi \vee \beta\}$. He is able to delete m individual judgment sets from the profile $\mathbf{T}$.

Since no judge accepts β, it will never be in the collective outcome. Therefore, the new agreement of the desired set with the new outcome has to occur in the conclusion. To include α_0, the chair has to delete m judges to lower the acception threshold to $n + 1$. These judges' individual judgment sets have to be deleted from $\mathbf{T_2}$ so that γ loses m affirmations and is not contained in the collective outcome anymore. If some x_i is not contained in one of the sets C_j that match the individual judgment sets of the deleted judges, the corresponding α_i loses too many affirmations and is therefore rejected in the new collective outcome. The control action is successful (i.e., $\varphi \vee \beta$ is contained in the new collective outcome) if and only if the sets C_j corresponding to the deleted individual judgment sets form an exact cover of X. This shows that $UPQR_{1/2}$-CR-Possible-Control-by-Deleting-Judges is NP-complete.

To prove the second part, we create a new agenda Φ' from Φ by removing β, $\varphi \vee \beta$, and the corresponding negations, and by adding the formula $\psi = (\neg\alpha_0 \wedge \gamma) \vee (\alpha_0 \wedge \neg\gamma)$ and its negation. Let $\mathbf{T'} = \mathbf{T'_1} \cup \mathbf{T'_2}$ be the resulting profile that is obtained by restricting $\mathbf{T_1}$ and $\mathbf{T_2}$ to Φ' and by adding the corresponding conclusions to all J_i and L_j. Then it holds that

$$UPQR_{1/2}(\mathbf{T'}) = \{\neg\alpha_0, \alpha_1, \ldots, \alpha_{3m}, \gamma, \psi\}.$$

Let $J' = \{\alpha_0, \alpha_1, \ldots, \alpha_{3m}, \neg\gamma, \psi\}$ and let the chair be able to delete m judgment sets. To preserve the agreement on the conclusion, the chair has to change the collective outcome in regard to α_0 as well as γ. Following the argumentation above, the chair has to delete exactly m judgment sets, can only delete judgment sets from $\mathbf{T_2}$ and therefore only preserves the agreements concerning the α_i if and only if the sets C_j corresponding to the deleted individual judgment sets form an exact cover of X. Thus $UPQR_{1/2}$-CR-NECESSARY-CONTROL-BY-DELETING-JUDGES is NP-complete. □

We now turn to control by replacing judges.

Theorem 9. *$UPQR_q$-CR-*POSSIBLE-CONTROL-BY-REPLACING-JUDGES *and $UPQR_q$-CR-*NECESSARY-CONTROL-BY-REPLACING-JUDGES *both are* NP*-complete for each rational quota q, $0 \leq q < 1$, even for a complete desired set.*

Proof. The proof works by a reduction from the problem DOMINATING-SET. Let (G, k) with $G = (V, E)$ and $V = \{v_1, \ldots, v_n\}$ be a DOMINATING-SET instance. The neighbors of vertex v_i (including v_i itself) will be denoted by $v_i^1, v_i^2, \ldots, v_i^{j_i}$ for some j_i.

For the first part of the theorem (i.e., for showing NP-completeness of $UPQR_q$-CR-POSSIBLE-CONTROL-BY-REPLACING-JUDGES), first assume that the quota q is lower than $1/2$. We construct an instance of the control problem as follows. The agenda Φ contains the literals $v_1, \ldots, v_n$, β, γ, the formula $\psi \vee \beta$, where $\psi = \varphi_1 \wedge \cdots \wedge \varphi_n \wedge \gamma$ and $\varphi_i = v_i^1 \vee \cdots \vee v_i^{j_i}$, and all corresponding negations. The profile $\mathbf{J} = (J_1, \ldots, J_m)$ with $m = 2k + 1$ judges, the outcome, and the chair's desired set J can be seen in Table 3(a).

The chair can choose at most k judgment sets from the profile $\mathbf{K} = (K_1, \ldots, K_n)$ with $K_i = \{\neg\beta, \neg\gamma, v_i, \neg v_j, \neg(\psi \vee \beta) \mid 1 \leq j \leq n, i \neq j\}$ to replace judgment sets in $\mathbf{J}$. The formula β will never be contained in the outcome because no judge accepts it. In order to achieve the desired additional agreement between the new outcome and J, the chair has to get the conclusion and therefore ψ accepted. Each v_i needs exactly one additional affirmation to be contained in the new outcome. Note that only judgment sets in the third block can be replaced (or else γ would lose an affirmation, would not be contained in the collective outcome anymore, and thus ψ cannot be evaluated to true). Since $\psi \vee \beta$ is contained in the new outcome if and only if the accepted v_i form a dominating set, and since only k judgment sets can be replaced, the control action is successful under closeness-respecting preferences if and only if G has a dominating set of size k.

In the case of a quota q greater than or equal to $1/2$, the agenda changes slightly. Instead of the formula $\psi \vee \beta$ and its negation the new agenda Φ' contains the formula $\psi' \vee \neg\beta$ with $\psi' = \varphi_1' \wedge \cdots \wedge \varphi_n' \wedge \neg\gamma$ and $\varphi_i' = \neg v_i^1 \vee \cdots \vee \neg v_i^{j_i}$, and its negation, $\neg(\psi' \vee \neg\beta)$. The profile $\mathbf{J'} = (J_1', \ldots J_m')$ with $m = 2k + 1$ judges, the outcome, and the chair's desired set J' can be seen in Table 3(b).

Let $\mathbf{K'} = (K_1', \ldots, K_n')$ be a profile, where

$$K_i' = \{\beta, \gamma, \neg v_i, v_j, \neg(\psi' \vee \neg\beta) \mid 1 \leq j \leq n, i \neq j\}$$

Table 3. Construction for the first part of the proof of Theorem 9

(a) Rational quota q with $0 \leq q < 1/2$

Judgment set	v_1	$\cdots$	v_n	β	γ	$\psi \vee \beta$
$J_1, \ldots, J_{\lfloor m \cdot q \rfloor}$	1	$\cdots$	1	0	1	1
$J_{\lfloor m \cdot q \rfloor + 1}$	0	$\cdots$	0	0	1	0
$J_{\lfloor m \cdot q \rfloor + 2}, \ldots, J_m$	0	$\cdots$	0	0	0	0
$UPQR_q$	0	$\cdots$	0	0	1	0
J	0	$\cdots$	0	1	1	1

(b) Rational quota q with $1/2 \leq q < 1$

Judgment set	v_1	$\cdots$	v_n	β	γ	$\psi' \vee \neg\beta$
$J'_1, \ldots, J'_{\lceil m \cdot (1-q) \rceil - 1}$	0	$\cdots$	0	1	0	1
$J'_{\lceil m \cdot (1-q) \rceil}$	1	$\cdots$	1	1	1	0
$J'_{\lceil m \cdot (1-q) \rceil + 1}, \ldots, J'_m$	1	$\cdots$	1	1	1	0
$UPQR_q$	1	$\cdots$	1	1	0	0
J'	1	$\cdots$	1	0	0	1

for $1 \leq i \leq n$. Again, the chair is able to replace k judgment sets from $\mathbf{J}'$ with k judgment sets from $\mathbf{K}'$. A formula needs at least $\lceil m(1-q) \rceil$ rejections in order to not be accepted. Since every judge accepts β, its negation will never be contained in the collective outcome. Thus the chair has to get ψ' accepted so as to achieve the desired additional agreement of the new outcome with J'. The argumentation then follows the first case: Since ψ' is true if and only if the rejected v_i form a dominating set and since the k replaceable judgment sets must be from the third block, the control action is successful under closeness-respecting preferences if and only if G has a dominating set of size k.

We prove the second part of the theorem (i.e., NP-completeness of $UPQR_q$-CR-Necessary-Control-by-Replacing-Judges) in a similar way. Unlike in the first part of the proof, the chair now has to necessarily prefer the new outcome to the actual one. That means that all existing agreements have to be preserved. Remove β from the former agenda Φ (respectively, Φ') and replace all appearances of ψ (respectively, ψ') with the formula $\Psi = \psi \vee (\neg v_1 \wedge \cdots \wedge \neg v_n)$ (respectively, $\Psi' = \psi' \vee (v_1 \wedge \cdots \wedge v_n)$). All required changes in the profiles $\mathbf{J}^*$ (respectively, $\mathbf{J}'^*$), the outcomes, and the desired sets J^* (respectively, J'^*) can be seen in Table 4(a) (respectively, in Table 4(b)).

To obtain the profiles $\mathbf{K}^*$ (respectively, $\mathbf{K}'^*$) of judgment sets to choose from, the premises of the judgment sets in $\mathbf{K}$ (respectively, $\mathbf{K}'$) restricted to the corresponding new agenda remain unchanged and the new conclusion is evaluated accordingly. As above the chair is allowed to replace k judgment sets. The chair has to change some premise different from γ in order to achieve a new

Table 4. Construction for the second part of the proof of Theorem 9

(a) Rational quota q with $0 \leq q < 1/2$

Judgment set	$v_1 \cdots v_n$ γ	Ψ
$J_1^*, \ldots, J_{\lfloor m \cdot q \rfloor}^*$	$1 \cdots 1$ 1	1
$J_{\lfloor m \cdot q \rfloor + 1}^*$	$0 \cdots 0$ 1	1
$J_{\lfloor m \cdot q \rfloor + 2}^*, \ldots, J_m^*$	$0 \cdots 0$ 0	1
$UPQR_q$	$0 \cdots 0$ 1	1
J^*	$1 \cdots 1$ 1	1

(b) Rational quota q with $1/2 \leq q < 1$

Judgment set	$v_1 \cdots v_n$ γ	Ψ'
$J_1'^*, \ldots, J_{\lceil m \cdot (1-q) \rceil - 1}'^*$	$0 \cdots 0$ 0	1
$J_{\lceil m \cdot (1-q) \rceil}'^*$	$1 \cdots 1$ 0	1
$J_{\lceil m \cdot (1-q) \rceil + 1}'^*, \ldots J_m'^*$	$1 \cdots 1$ 1	1
$UPQR_q$	$1 \cdots 1$ 0	1
J'^*	$0 \cdots 0$ 0	1

agreement. But after this action the second part of Ψ (respectively, Ψ') is not satisfied anymore. In order to preserve the agreement of the outcome with her desired set regarding the conclusion, the chair has to replace the judgment sets from the third block with the judgment sets from $\mathbf{K}^*$ (respectively, $\mathbf{K}'^*$) that correspond to the vertices in a dominating set of G. It follows that the control action is successful if and only if G has a dominating set of size k. □

Finally, we turn to unrestricted and top-respecting preferences.

Proposition 10. *Let $\mathscr{C}$ be one of the control types* Adding-Judges*,* Deleting-Judges*, and* Replacing-Judges*, let $T \in \{U, TR\}$ be a preference type, and let the desired set be complete. For each rational quota q, $0 \leq q < 1$, $UPQR_q$-T-*Possible-Control-by-$\mathscr{C}$ *is in* P*.*

Proof. In the case of unrestricted preferences, the chair possibly prefers every new outcome to the actual outcome. Since her desired set is complete, she only has to check if she can change a premise so as to change the collective judgment set. This is possible in polynomial time for every $\mathscr{C}$.

In the case of top-induced preferences, the chair possibly prefers every new outcome to the actual outcome as long as the latter is not identical to her desired set. Therefore, it also suffices to change some premise if possible. □

Proposition 11. *Let $\mathscr{C}$ be a control type. For each rational quota q*, $0 \leq q < 1$, *$UPQR_q$-U-* Necessary-Control-by- $\mathscr{C}$ *is possibly immune.*

Proof. In the case of unrestricted preferences, the collective judgment set is always possibly preferred to every other judgment set that can occur as a new outcome after the control action. □

4 Bribery

In this section, we study the complexity of bribery problems related to the types of preferences defined in Sect. 2. Bribery in judgment aggregation has been introduced by Baumeister et al. [5,6]; however, their results are restricted to Hamming-distance-respecting preferences and to the premise-based procedure only.

4.1 Preliminaries

We now formally define the relevant bribery problems for the uniform premise-based quota rule with quota q and for some given preference type T.

$UPQR_q$-T-Possible-Bribery	
Given:	An agenda Φ, a profile $\mathbf{J} \in \mathscr{J}(\Phi)^n$, a desired set J, and a positive integer k (the budget).
Question:	Is there a new profile $\mathbf{J}' \in \mathscr{J}(\Phi)^n$ with at most k changed individual judgment sets so that $UPQR_q(\mathbf{J}') \succ UPQR_q(\mathbf{J})$ for some $\succsim \in T_J$?

Concerning the analogous problem $UPQR_q$-T-Necessary-Bribery, the condition $UPQR_q(\mathbf{J}') \succ UPQR_q(\mathbf{J})$ is required to hold for *all* $\succsim \in T_J$. In the two corresponding microbribery problems (also introduced by Baumeister et al. [5,6]), the briber is allowed to change k premises instead of k whole judgment sets.

Given an agenda Φ, a profile $\mathbf{J} \in \mathscr{J}(\Phi)^n$, a desired set J, and a positive integer k, in the exact variant of the bribery (respectively, microbribery) problem we ask whether the briber can change up to k individual judgment sets (respectively, premises) such that $J \subseteq UPQR_q(\mathbf{J}')$, where $\mathbf{J}'$ denotes the modified profile. We denote these problems by $UPQR_q$-Exact-Bribery and $UPQR_q$-Exact-Microbribery. Again, it is easy to see that all decision problems in this section are in NP.

Definition 12. *Let Φ be an agenda and let $\mathscr{B}$ be a given bribery type. A resolute judgment aggregation procedure F is* necessarily/possibly immune to $\mathscr{B}$ *for induced preferences of type $T \in \{U, TR, CR\}$ if for all profiles $\mathbf{J}$ and for each desired set J, the briber necessarily/possibly weakly prefers the outcome $F(\mathbf{J})$ to the outcome $F(\mathbf{J}')$ for type T_J, where $\mathbf{J}'$ denotes the new profile after bribery of type $\mathscr{B}$ has been exerted.*

4.2 Results for Bribery

We now present our results for bribery in judgment aggregation.

Theorem 13. *For each rational quota q, $0 \leq q < 1$,*

1. $UPQR_q$-Exact-Bribery $\leq_m^p$ $UPQR_q$-T-Possible-Bribery *for each preference type $T \in \{U, TR, CR\}$;*
2. $UPQR_q$-Exact-Microbribery $\leq_m^p$ $UPQR_q$-T-Possible-Microbribery *for each preference type $T \in \{U, TR, CR\}$.*

The simple proof (an adaption of the proof of Proposition 6) uses an incomplete desired set, so we again have to consider the case of complete desired sets separately.

Baumeister et al. [5] show NP-completeness of exact bribery (respectively, microbribery) with an incomplete desired set for the premise-based procedure (*PBP*), which—recall—is a special case of $UPQR_{1/2}$ for an odd number of judges. Their proofs can be modified so as to work for every rational quota q with $0 \leq q < 1$ and for every number of judges. For a complete desired set and *PBP*, Baumeister et al. [5] prove that the exact bribery problem remains NP-complete and provide a P algorithm that solves the exact microbribery problem. The P algorithm for exact microbribery can also easily be adapted to work for every rational quota q with $0 \leq q < 1$ and for every number of judges. Since under top-respecting preferences the briber necessarily prefers only her desired set to any other possible outcome and assuming that the briber's desired set is complete, we thus have that $UPQR_{1/2}$-*TR*-Necessary-Bribery is NP-complete, but for each rational quota q, $0 \leq q < 1$, $UPQR_q$-*TR*-Necessary-Microbribery is in P.

Next we consider closeness-respecting preferences for bribery problems.

Theorem 14. *For each rational quota q, $0 \leq q < 1$, $UPQR_q$-CR-*Necessary-Bribery *and $UPQR_q$-CR-*Possible-Bribery *both are* NP-*complete, even for a complete desired set.*

Proof. The proof works by a reduction from the problem Restricted-SAT (defined in Sect. 2) and adapts an idea of Endriss et al. [9]. We first show that $UPQR_q$-*CR*-Necessary-Bribery is NP-complete. Let φ be a Restricted-SAT instance. For a quota q equal to or greater than $1/2$, the agenda Φ contains the variables of φ (i.e., $\alpha_1, \dots, \alpha_m$), a literal β, the formula $\psi \vee \beta$ with $\psi = \varphi \vee (\neg\alpha_1 \wedge \cdots \wedge \neg\alpha_m)$ and all corresponding negations. Let m be the briber's budget, let $n = 2m + 1$ be the number of judges, and let $J = \{\alpha_1, \dots, \alpha_m, \beta, \psi \vee \beta\}$ be the briber's desired set. The profile $\mathbf{J}$ is shown in Table 5(b).

Even by changing m judgments β will never be in the collective judgment set. Therefore, at least one α_i has to be set to 1 to obtain the required additional agreement with J. This is possible because every α_i can be included in the new outcome by changing exactly m judgment sets in the second block of judges. Since the agreement with $\psi \vee \beta$ has to be preserved, the bribery is successful under closeness-respecting preferences if and only if φ has a satisfying assignment.

In the case of $0 \leq q < 1/2$, the agenda has to be slightly changed. The formula $\psi \vee \beta$ and its negation are replaced by the formula $\psi \vee \neg\beta$ and its negation. The corresponding profile $\mathbf{J}' = (J'_1, \dots, J'_n)$ is shown in Table 5(a). Since it is impossible for the briber to reject β and since all agreements of the collective outcome with J' have to be preserved, the bribery is successful under closeness-respecting preferences if and only if φ has a satisfying assignment.

We now turn to the second part of the theorem (i.e., to NP-completeness of $UPQR_q$-*CR*-Possible-Bribery). This can be shown in a similar way. Change the agenda described above by replacing the formula $\psi \vee \beta$ with $\varphi \vee \beta$ in the first case (respectively, $\psi \vee \neg\beta$ with $\varphi \vee \neg\beta$ in the second case) including all corresponding negations. Let $\mathbf{J}^*$ (respectively, $\mathbf{J}'^*$) be the profile concerning the

Table 5. Construction for the proof of Theorem 14

(a) Rational quota q with $0 \leq q < 1/2$

Judgment set	α_1	$\cdots$	α_m	β	$\psi \vee \neg\beta$
$J'_1, \ldots, J'_n$	0	$\cdots$	0	1	1
$UPQR_q$	0	$\cdots$	0	1	1
J'	1	$\cdots$	1	0	1

(b) Rational quota q with $1/2 \leq q < 1$

Judgment set	α_1	$\cdots$	α_m	β	$\psi \vee \beta$
$J_1, \ldots, J_{\lfloor n \cdot q \rfloor - (m-1)}$	1	$\cdots$	1	0	0
$J_{\lfloor n \cdot q \rfloor - (m-1)+1}, \ldots, J_n$	0	$\cdots$	0	0	1
$UPQR_q$	0	$\cdots$	0	0	1
J	1	$\cdots$	1	1	1

corresponding new agenda with the premises of the individual judgment sets as seen in the corresponding part of Table 5 and the conclusions evaluated accordingly. Note that the collective outcomes only differ in the conclusion, which is rejected in both cases. Further, let $J^* = \{\neg\alpha_1, \ldots, \neg\alpha_m, \beta, \varphi \vee \beta\}$ (respectively, $J'^* = \{\neg\alpha_1, \ldots, \neg\alpha_m, \neg\beta, \varphi \vee \neg\beta\}$) be the briber's desired set. Since the only additional agreement the briber can achieve is the conclusion, similar arguments as above complete the proof. □

We now handle the case of microbribery for closeness-respecting preferences.

Theorem 15. *For each rational quota q, $0 \leq q < 1$, $UPQR_q$-CR-*Necessary-Microbribery *and $UPQR_q$-CR-*Possible-Microbribery *both are* NP-*complete, even for a complete desired set.*

Proof. This proof works similarly to the proof of Theorem 14. For proving the first part (NP-completeness of $UPQR_q$-CR-Necessary-Microbribery), the only change is the number of judges in the different blocks of judges: Judges $1, \ldots, \lfloor n \cdot q \rfloor$ form the first block, while the second block consists of judges $\lfloor n \cdot q \rfloor + 1, \ldots, n$. Then a similar argumentation as in the proof of Theorem 14 applies. Note that the briber is only allowed to change k premises instead of k whole individual judgment sets.

For the proof of the second part (i.e., for showing NP-completeness of $UPQR_q$-CR-Possible-Microbribery), we use the agendas from the corresponding parts of the proof of Theorem 14. In the first case, judges $1, \ldots, \lfloor n \cdot q \rfloor$ accept all premises but β and reject the conclusion, judges $\lfloor n \cdot q \rfloor + 1, \ldots, n$ reject all formulas, the collective outcome contains all negated formulas, and the briber accepts only β and $\varphi \vee \beta$ and rejects the remaining propositions. In the second case, each appearance of β or $\neg\beta$ in the first case is replaced with its

Table 6. Overview of results for $UPQR_{1/2}$-T-POSSIBLE/NECESSARY-CONTROL-BY-$\mathscr{C}$ for $\mathscr{C} \in$ {ADDING-JUDGES, DELETING-JUDGES, REPLACING-JUDGES} and $T \in \{U, TR, CR\}$

		U	TR	CR
Incomplete DS	POSSIBLE	NP-complete	NP-complete	NP-complete
	NECESSARY	possibly immune	NP-complete	NP-complete
Complete DS	POSSIBLE	P	P	NP-complete
	NECESSARY	possibly immune	NP-complete	NP-complete

Table 7. Overview of results for $UPQR_q$-T-POSSIBLE/NECESSARY-BRIBERY/ MICROBRIBERY for $T \in \{U, TR, CR\}$

		U	TR	CR
Incomplete DS	POSSIBLE	NP-complete	NP-complete	NP-complete
	NECESSARY	possibly immune	NP-complete	NP-complete
Complete DS	POSSIBLE	P	P	NP-complete
	NECESSARY	possibly immune	NP-complete[2]/P	NP-complete

complement. Once again, the bribery action is successful if and only if φ has a satisfying assignment. □

The following propositions can be proven in the same way as Propositions 10 and 11.

Proposition 16. *Let $T \in \{U, TR\}$ be a preference type, and let the briber's desired set be complete. For each rational quota q, $0 \le q < 1$, $UPQR_q$-T-*POSSIBLE-BRIBERY *and $UPQR_q$-T-*POSSIBLE-MICROBRIBERY *both are in* P.

Proposition 17. *For each rational quota q, $0 \le q < 1$, $UPQR_q$-U-*NECESSARY-BRIBERY *and $UPQR_q$-U-*NECESSARY-MICROBRIBERY *both are possibly immune.*

5 Conclusions and Future Work

We have studied bribery and microbribery as well as three types of control in judgment aggregation. While these problems were introduced in previous work by Baumeister et al. [3,5,6], they have been studied only for Hamming-distance-respecting preferences so far. Our contribution is to extend this study to the case of more general preference notions, including closeness-respecting and top-respecting preferences that are due to Dietrich and List [8] and have been applied to manipulation in judgment aggregation by Baumeister et al. [4,5]. Furthermore, our results for bribery and microbribery apply to uniform premise-based quota rules that generalize the premise-based procedure. An overview of our complexity results is given in Table 6 for control and in Table 7 for bribery and microbribery. Here, DS stands for "desired set." For control by replacing judges, NECESSARY-CONTROL-BY-ADDING/DELETING-JUDGES for unrestricted preferences, and POSSIBLE-CONTROL-BY-ADDING/DELETING-JUDGES for a complete desired set inducing unrestricted and top-respecting preferences, the results are shown for a general rational quota q, $0 \leq q < 1$. The results for bribery and microbribery are identical, except for the NECESSARY-BRIBERY/MICROBRIBERY problem where we have a complete desired set inducing top-respecting preferences. The entry NP-complete/P here means that this problem is NP-complete for bribery[2] but in P for microbribery.

Regarding Hamming-distance-respecting preferences, note that Baumeister et al. [2,6] have already studied the complexity of bribery and microbribery with *incomplete* desired sets and for the *premise-based procedure* only. However, their proofs can easily be adapted to also apply to complete desired sets and to uniform premise-based quota rules. Similarly, some of the results of Baumeister et al. [2,3] for exact control and for control problems under Hamming-distance-respecting preferences apply to incomplete desired sets only, but the proofs only have to be slightly adapted to work for the case of complete desired sets, too.

Regarding future work, we propose to study the complexity of these problems for different families of judgment aggregation procedures, to study other preferences for the attacker (e.g., by using other distance measures), and to study the complexity of control by bundling judges introduced by Baumeister et al. [4].

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