Verification in Argument-Incomplete Argumentation Frameworks

Dorothea Baumeister, Jörg Rothe, and Hilmar Schadrack(✉)

Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany
{baumeister,rothe,schadrack}@cs.uni-duesseldorf.de

Abstract. Incomplete knowledge in argumentation frameworks may occur during the single steps of an elicitation process, when merging different beliefs about the current state of an argumentation framework, or when it is simply not possible to obtain complete information. The semantics of argumentation frameworks with such incomplete knowledge have previously been modeled in terms of an inco mplete attack relation among the given arguments by Cayrol et al. [12] or when adding an argument that interacts with already present arguments [14]. We propose a more general model of argument-incomplete argumentation frameworks with a variable set of arguments, and we study the related verification problems for various semantics in terms of their computational complexity.

1 Introduction

A discussion between human beings is a form of communicating opinions and thoughts about a given subject. These opinions and their interactions are often highly complex and thus hard to formalize mathematically. The goal of abstract argumentation is to model discussions between (human or software) agents by abstracting from the actual content of arguments and from the reasons of why they attack each other, and instead to consider a given set of arguments along with an attack relation on it and to find certain subsets of the arguments that fulfill certain justification criteria. In 1995, Dung [18] introduced a formal model to describe discussions abstractly. His model uses a graph structure where the nodes represent arguments, and the attacks between arguments are modeled through directed edges. He also introduced various semantics, i.e., criteria that can express different kinds and levels of justification for certain subsets of the arguments. His highly influential model has been used by many researchers, who developed additional ideas of how to extend it so as to make it an elegant, rich, and attractive model for abstract group argumentation. We refer the reader to the book by Rahwan and Simari [30] for more background on abstract argumentation in artificial intelligence.

In this paper, we develop a new model for argumentation frameworks on the basis of Dung's work [18], namely, *argument-incomplete argumentation frameworks*. Our goal is to extend the standard model by allowing uncertainty over the set of arguments. In our model, we have a set of arguments that already

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are known to exist, and another set of arguments that in addition contains those arguments which might become relevant in a later state of the discussion. We study properties and semantics, namely conflict-freeness, admissibility, preferredness, stability, completeness, and groundedness, suitably adapted to the argument-incomplete setting, where we distinguish between properties holding possibly and necessarily. Besides the formal model, the main contribution of this work are complexity results of appropriately extended variants of the verification problem [19], asking whether or not a given set of arguments will fulfill a previously specified property either possibly or necessarily.

Related Work and Motivation: In real-world discussions we cannot assume to know all arguments or attacks in advance, or how important they are for the discussion, or to fully capture the dynamics of a discussion. Therefore, we are trying to take an early step to model situations in which complete information is not available by allowing the set of arguments to be uncertain. This can happen, for example, in a well developed discussion in which many, or even all, possible arguments are known already but where certain external limitations can change, which may have an impact on the validity or importance of the arguments. It may be safe to assume that some of the arguments are always valid, but which of the uncertain arguments are valid may depend on the circumstances. For example, if the citizens of a town discuss whether a public swimming pool, an opera house, or a library are to be built, it will have an impact on some of the arguments if it suddenly turns out that the budget deficit is higher than expected—some arguments may then be invalid or less important, while others remain valid and crucial. It would be interesting to know which sets of arguments (possibly also containing uncertain arguments) are justified for different limitations.

As another example, consider the case of different knowledge bases of agents. All the agents share the same set of possible arguments, but they disagree on their importance. Hence, every agent has her own "belief stage" resulting in different individual views on the argumentation framework. Such belief-staged argumentation frameworks can be modeled by an argument-incomplete argumentation framework, and so can the aggregated opinion of the agents, obtained by agreeing on some arguments to be important, leaving the others as uncertain.

Modeling discussions via argument-incomplete argumentation frameworks may help to answer the question of whether it is possible to make early decisions about which sets of arguments will fulfill certain criteria possibly (i.e., in at least one way regarding currently uncertain arguments that may arise—or turn out to be important—in the future) or necessarily (i.e., in any way regarding currently uncertain arguments that may arise—or turn out to be important—in the future).

The need for a model that is capable of capturing these ideas is also motivated by the interdisciplinary graduate school "Online Participation"[1] run by Heinrich-Heine-Universität Düsseldorf in cooperation with Fachhochschule für öffentliche Verwaltung and with a number of practice partners and municipal councils. Researchers from the social sciences, political science, communication science,

[1] We refer to the website http://www.fortschrittskolleg.de for further details.

and computer science are involved in creating a broad knowledge base about how to discuss in online platforms, as well as in designing a software tool for performing this in practice. This work aims to provide a solid theoretical foundation.

Incomplete argumentation frameworks have been introduced by Cayrol et al. [12], who define so-called "partial argumentation frameworks" by distinguishing the attacks into those that are definitely part of the argumentation framework, those that are definitely not part of the argumentation framework, and those that are not certain—but possible—to occur. They further define a *completion* of such a partial argumentation framework as a standard argumentation framework that contains all the arguments of the partial argumentation framework, at least the attacks definitely contained, and maybe also some of the possible attacks. We use a similar idea in our model of argument-incomplete argumentation frameworks.

The model of Cayrol et al. [13,14] to study changes in the argument set has a different goal than our model. They introduce change operations that allow for the addition or deletion of one attack, or one argument together with a set of attacks regarding this argument. Their work focuses on a classification of how such changes can possibly alter the outcome.

Other approaches regarding changes in the argument set are due to Boella et al. [7], who define general principles for the abstraction of arguments and attacks, mainly for the grounded semantics. They address the question of which arguments or attacks can be removed such that the extensions remain unchanged.

In so-called "probabilistic argumentation frameworks" (introduced by Li et al. [26]; for more information, see the work of Doder and Woltran [17]), every argument and attack has an associated probability that yields the likelihood of that argument or attack to be part of an induced argumentation framework. This can be seen as an intermediate state between complete knowledge and incomplete information. Li et al. [26] show that computing the probability of a set of arguments being justified regarding a semantics can be intractable. Therefore, the authors approximate it by means of a Monte-Carlo simulation. Fazzinga et al. [20] show that this computation indeed is hard for, e.g., the complete, grounded, and preferred semantics, but is easy for stability and admissibility. They further discuss approximation algorithms [21].

Baumeister et al. [6] describe a model of *attack*-incomplete argumentation frameworks.

The idea of extending models of complete information to allow for incomplete information is not new; it has been applied, e.g., in the field of computational social choice, especially in voting theory (see, e.g., the book chapters by Boutilier and Rosenschein [8] and Baumeister and Rothe [1]). Konczak and Lang [23] introduced the notions of *possible* and *necessary winners* in elections, which then were studied in different variants (see, e.g., Konczak and Lang [23], Xia and Conitzer [33], Lang et al. [24], and Chevaleyre et al. [15]) and for different settings (see, e.g., the work of Baumeister et al. [2,3]). Other fields in which the notions of possibility and necessity are used include judgment aggregation [4], fair division [5,9–11], and algorithmic game theory [25].

This paper is structured as follows. In Sect. 2, we introduce the known model of abstract argumentation frameworks, and we provide the needed notions from complexity theory. In Sect. 3, we describe our model for attack-incomplete argumentation frameworks and present our results. Section 4, finally, gives our conclusions and comments on some open questions.

2 Preliminaries

In this section, we introduce formal definitions of the central notions related to (classical) argumentation frameworks. The basic ideas are due to Dung [18], and we will be using some notation from the book chapter by Dunne and Wooldridge [19].

An *argumentation framework* AF is a pair $\langle \mathscr{A}, \mathscr{R} \rangle$, where $\mathscr{A}$ denotes a set of arguments, and $\mathscr{R} \subseteq \mathscr{A} \times \mathscr{A}$ the attack relation. For every pair $(a, b) \in \mathscr{R}$ we say a *attacks* b, and for simplicity we often write $a \rightarrow b$. If $a \rightarrow b$ and $b \rightarrow a$, we simply write $a \leftrightarrow b$. Every argumentation framework $AF = \langle \mathscr{A}, \mathscr{R} \rangle$ can be seen as a directed Graph $G_{AF} = (V, E)$ by using the arguments as vertices and the attacks as edges, i.e., $V = \mathscr{A}$ and $E = \mathscr{R}$.

Before going into further detail of the abstract argumentation scheme by Dung, we will present an easy example, which will be used again later on.

Example 1. Assume we have seven arguments, $\{a, b, c, d, e, f, g\}$, and nine attacks: $a \rightarrow b, a \rightarrow c, b \rightarrow d, c \rightarrow d, e \rightarrow d, e \leftrightarrow f, e \rightarrow g$, and $g \rightarrow g$. Then the appropriate argumentation framework is

$$\begin{aligned} AF = \langle \mathscr{A}, \mathscr{R} \rangle = & \langle \{a, b, c, d, e, f, g, \}, \\ & \{(a, b), (a, c), (b, d), (c, d), (e, d), (e, f), (e, g), (f, e), (g, g)\} \rangle. \end{aligned}$$

The graph representation G_{AF} of this argumentation framework is shown in Fig. 1.

We now define properties in argumentation frameworks, mainly for sets of arguments. All of them were introduced by Dung [18]. We start with the three most basic properties: conflict-freeness, acceptability, and admissibility. Let $AF = \langle \mathscr{A}, \mathscr{R} \rangle$ be an argumentation framework.

- A set $S \subseteq \mathscr{A}$ is called *conflict-free* if there are no arguments $a, b \in S$ such that $a \rightarrow b$.
- An argument $a \in \mathscr{A}$ is called *acceptable with respect to* $S \subseteq \mathscr{A}$ if for all arguments $b \in \mathscr{A}$ with $b \rightarrow a$, we have at least one argument $c \in S$ such that $c \rightarrow b$.
- A conflict-free set $S \subseteq \mathscr{A}$ is called *admissible* if every argument $a \in S$ is acceptable with respect to S.

More advanced properties are preferredness, stability, completeness, and groundedness, and Dung calls them *semantics* in his work [18].

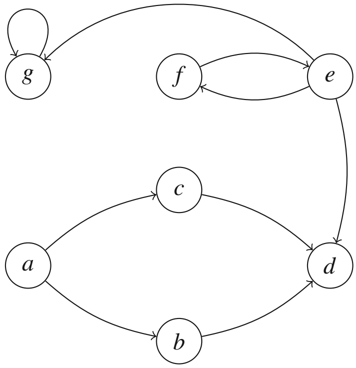


Fig. 1. The graph $G_{AF} = (\mathscr{A}, \mathscr{R})$ for the argumentation framework $AF = \langle \mathscr{A}, \mathscr{R} \rangle$ from Example 1

- A set $S \subseteq \mathscr{A}$ is called *preferred* if S is a maximal (with respect to set inclusion) admissible set.
- A set $S \subseteq \mathscr{A}$ is called *stable* if S is conflict-free and for all arguments $b \in \mathscr{A} \setminus S$, there is at least on argument $a \in S$ with $a \rightarrow b$.
- A set $S \subseteq \mathscr{A}$ is called *complete* if S is admissible and contains all arguments $a \in \mathscr{A}$ that are acceptable with respect to S.
- A set $S \subseteq \mathscr{A}$ is called *grounded* if S is the least (with respect to set inclusion) fixed point of the characteristic function of the argumentation framework. The *characteristic function* F_{AF} of the argumentation framework AF is a function $F_{AF} : 2^{\mathscr{A}} \rightarrow 2^{\mathscr{A}}$ defined by

$$F_{AF}(S) = \{a \in \mathscr{A} \mid a \text{ is acceptable with respect to } S\}.$$

The characteristic function is monotonic with respect to set inclusion, and there always is exactly one least fixed point. Hence, there always is exactly one grounded set for a given argumentation framework. Additionally, starting with an arbitrary subset of the arguments, there always is an $i \in \mathbb{N}$ such that the i-fold composition of the characteristic function has a fixed point. All those fixed points are exactly the complete sets of the given argumentation framework.

Dung [18] shows how those above defined properties are related to each other. In particular, he proved that there always is a preferred set, and that every admissible set is a subset of a preferred set, every stable set is preferred, and every preferred set is complete. As already mentioned, there is exactly one grounded set in a given argumentation framework, and it is obviously complete. It is easy to see that a stable or a preferred set can be the grounded set, but there are argumentation frameworks in which the grounded set is neither preferred nor stable. Finally, every complete set is admissible and every admissible set is conflict-free, due to their definitions. Figure 2 summarizes these results.

In this work, we will focus on the six properties for sets of arguments introduced above. In the literature, conflict-freeness and admissibility are not called

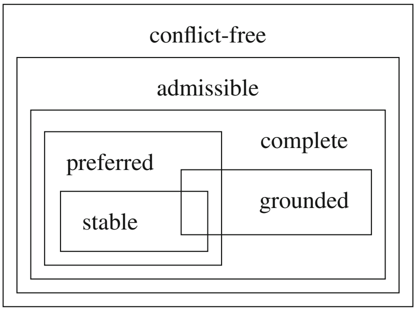


Fig. 2. A summary over how the different properties for sets of arguments used in this paper are correlated

semantics, as they are often considered to be basic conditions. However, for the sake of simplicity, we will call all these properties semantics.

Dung [18] also introduced the term "extension". For a given argumentation framework AF and a semantics **s**, a subset S of the arguments is called **s** *extension of* AF if S fulfills the conditions of semantics **s**.

We will now illustrate the above semantics in an example.

Example 2. Consider again the argumentation framework from Example 1, illustrated in Fig. 1. The only stable extension in this argumentation framework is $\{a, e\}$. Besides this, there is another preferred extension, namely $\{a, d, f\}$. There are no more preferred extensions. The unique grounded extension is $\{a\}$, but it is neither preferred nor stable. Besides those three extensions, there are no further sets that are complete. The only other admissible sets are $\emptyset$, $\{e\}$, $\{f\}$, and $\{a, f\}$. g is not part of any extension, as the self-attack always yields a conflict, and b or c is never part of any admissible set, because argument a which attacks them is never attacked itself.

We will now briefly mention the notions from complexity theory that we use in this paper. We assume the reader to be familiar with the basic notions, like the complexity classes P, NP, and coNP, as well as hardness, completeness, polynomial-time many-one-reducibility, $\leq_{\mathrm{m}}^{\mathrm{p}}$, and the notion of (oracle) Turing machines. The complexity class DP was introduced by Papadimitriou and Yannakakis [29] as the class of the differences of two NP problems. DP is also the second level of the boolean hierarchy over NP. $\Sigma_2^p = \mathrm{NP}^{\mathrm{NP}}$ contains those problems that are solvable by a nondeterministic oracle Turing machine with access to an NP oracle, and was introduced, together with $\Pi_2^p = \mathrm{coNP}^{\mathrm{NP}}$, by Meyer and Stockmeyer [27,31] as the second level of the polynomial hierarchy. It is known that $\mathrm{P} \subseteq \mathrm{NP} \subseteq \mathrm{DP} \subseteq \Sigma_2^p$, but it is not known whether any of these inclusion is strict. For further details, see [28].

Dunne and Wooldridge [19] give an overview over decision problems defined for argumentation frameworks. Among others, they investigate Verification, Credulous-Acceptance, Skeptical-Acceptance, Existence, and Nonemptiness for several semantics. Many of those decision problems are hard to decide, as they are complete for NP, coNP, DP, or even Π_2^p. We will focus on VERIFICATION (while we are going to change their notation slightly), which is easy to decide for all semantics studied here, except for preferredness for which it is known to be coNP-complete [16]. All easiness results follow immediately from the work of Dung [18].

s-VERIFICATION
Given: An argumentation framework $\langle \mathscr{A}, \mathscr{R} \rangle$ and a subset $S \subseteq \mathscr{A}$.
Question: Is S an **s** extension of AF?

The boldfaced letter **s** is a placeholder for any of the six semantics defined above. For better readability, we will use CF for *conflict-freeness*, AD for *admissibility*, PR for *preferredness*, ST for *stability*, CP for *completeness*, and GR for *groundedness*.

3 Argument-Incomplete Argumentation Frameworks

We now turn to our model extending classical argumentation frameworks: argument-incomplete argumentation frameworks. Cayrol et al. [12] introduced a model of incomplete argumentation framework in which the exact attacks are unknown. Cayrol et al. [14] discuss a model that allows for one of the following four so-called *change operations*: The addition (1) or deletion (2) of one attack, the addition of one argument together with at least one attack regarding this argument (3), and the deletion of one argument together with all corresponding attacks (4). Their goal is to find different classifications for how the set of *all extensions* (of a given semantics and argumentation framework) alter after applying *one* change. In contrast, we want to *verify* those *sets of arguments* (of a given semantics and argumentation framework) that can *become once* or *remain always* extensions for *arbitrarily many* changes in the argument set. After describing our model, we will study the computational complexity of the verification problem for various semantics in argument-incomplete argumentation frameworks.

3.1 Model

In our setting we do not know exactly which arguments will be part of the final discussion, but we know a set of arguments that are important already, and have a vague idea of which arguments may be important in the future. Formally, our model is defined as follows.

Definition 1. *An* argument-incomplete argumentation framework *is a triple* $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$*, where* $\mathcal{A}'$ *and* $\mathcal{A}$ *with* $\mathcal{A}' \subseteq \mathcal{A}$ *are sets of arguments, and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is an attack relation.*

The arguments in $\mathscr{A}'$ are those arguments that definitely are already part of the discussion. $\mathscr{A}$ contains, additionally to the arguments in $\mathscr{A}'$, also those arguments that could possibly join the discussion in the future.

Definition 2. *Let $IAF = \langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ be an argument-incomplete argumentation framework. For a set $\mathscr{A}^*$ of arguments with $\mathscr{A}' \subseteq \mathscr{A}^* \subseteq \mathscr{A}$, define the* restriction of *$\mathscr{R}$ to $\mathscr{A}^*$ by*

$$\mathscr{R}|_{\mathscr{A}^*} = \{(a, b) \in \mathscr{R} \mid a, b \in \mathscr{A}^*\}.$$

$AF^* = \langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ is called a completion of *IAF.*

Why is it plausible to assume that the number of possible arguments is finite? An answer to this question is that in real-world applications it is relatively safe to assume that the number of participants that are part of an discussion is limited, and that no individual has an infinite number of ideas to propose as arguments. In such scenarios, the total number of arguments must be finite. Also, why is it plausible to assume that all arguments are known in advance? Because in real-world applications arguments that do not have enough support by the participants are not important for the discussion. As soon as a new argument is introduced, however, we can ask how this argument is related to the existing arguments, and thus learn step by step new arguments and attacks once they come to life and then maybe become significant.

We now extend the definitions for classical argumentation frameworks to argument-incomplete ones, distinguishing between properties holding either *possibly* or *necessarily*.

Definition 3. *Let $IAF = \langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ be an argument-incomplete argumentation framework. For a property* $\mathbf{s}$*, call a subset $S \subseteq \mathscr{A}$ of arguments*

- possibly $\mathbf{s}$ *in IAF if there is a completion $AF^* = \langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ of IAF such that $S|_{\mathscr{A}^*} = S \cap \mathscr{A}^*$ is* $\mathbf{s}$ *in AF^* and*
- necessarily $\mathbf{s}$ *in IAF if for all completions $AF^* = \langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ of IAF, $S|_{\mathscr{A}^*} = S \cap \mathscr{A}^*$ is* $\mathbf{s}$ *in AF^*.*

We call a set S a *possibly* (respectively *necessarily*) $\mathbf{s}$ *extension of IAF* if S is possibly (respectively necessarily) $\mathbf{s}$ in IAF.

Remark 1. The following concluding remarks hold for all argument-incomplete argumentation frameworks.

- The possible and necessary semantics inherit the correlations of the properties from Dung's model, i.e., for example, possible stability implies possible preferredness.
- There always is a possibly preferred extension and a possibly grounded extension.
- A possibly grounded extension is not unique, but there is at most one necessarily grounded extension.
- Let S be a possibly $\mathbf{s}$ extension of $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$, $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ a completion such that S is an $\mathbf{s}$ extension of $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$, and $a \in \mathscr{A} \smallsetminus \mathscr{A}^*$. Then $S \cup \{a\}$ is a possibly $\mathbf{s}$ extension of $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ as well. We call those possibly $\mathbf{s}$ extensions *trivial supersets (of S)*.

Example 3. Consider again the argumentation framework from Example 1 but assume that some arguments, namely b, e, and g, are not certain yet. Then we have the argument-incomplete argumentation framework

$$IAF = \langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle = \langle \{a, c, d, f\}, \{a, b, c, d, e, f, g\}, \\ \{(a, b), (a, c), (b, d), (c, d), (e, d), (e, f), (e, g), (f, e), (g, g)\}\rangle.$$

Figure 3 illustrates this argument-incomplete argumentation framework. Solid vertices represent members of $\mathscr{A}'$, while dotted vertices stand for elements of $\mathscr{A} \smallsetminus \mathscr{A}'$. Dashed arcs symbolize the incoming and outgoing attacks of the elements $\mathscr{A} \smallsetminus \mathscr{A}'$. Note that the attacks drawn as black arcs will always be in any completion, while those attacks (x, y) drawn as dashed arcs are part of a completion if and only if x and y both are arguments in that completion.

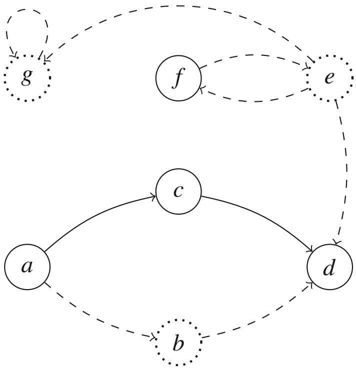


Fig. 3. The representation of the argument-incomplete argumentation framework $IAF = \langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ from Example 3

First, let us consider possibly **s** extensions of IAF. Obviously, all **s** extensions of the original argumentation framework AF are possibly **s** extensions of IAF. Hence, $\{a, e\}$ is possibly stable (and also possibly preferred, possibly complete, and possibly admissible). However, $\{a, d, f\}$ is also possibly stable, as it is stable in the completion $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$, and there are no other possibly stable nor possibly preferred sets, except for trivial supersets. $\{a\}$ is the grounded extension of AF, and it remains to be a possibly grounded extension of IAF, but additionally $\{a, d, f\}$ is possibly grounded, as it is the unique grounded extension of $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$. It is easy to see that there are no more possibly grounded extensions, and that the only possibly complete sets are $\{a\}$, $\{a, e\}$, and $\{a, d, f\}$ (both except for trivial supersets). Besides the admissible sets of AF, there is one more possibly admissible set of IAF (except for trivial supersets), namely $\{a, d\}$, which is admissible in, e.g., the completion $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$.

To find all necessarily **s** extensions of IAF, it is sufficient to check whether the possibly **s** extensions mentioned above, except for trivial supersets, are **s** extensions of all completions. Hence, there is no necessarily stable extension, as $\{a, e\} \cap \mathscr{A}'$ is not stable in $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$ and $\{a, d, f\}$ is not stable in $\langle \mathscr{A}, \mathscr{R} \rangle$, and the same completions also prevent $\{a\}$ and $\{a, d, f\}$ from being necessarily grounded. The only necessarily preferred extension is $\{a, d, f\}$, because $\{a, e\} \cap \mathscr{A}'$ is not preferred in $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$. $\{a\}$ and $\{a, e\}$ are not necessarily complete because of the completion $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$. Lastly, all possibly admissible sets except for $\{a, d\}$ are also necessarily admissible.

3.2 Possible and Necessary Verification

Using **s**-Verification as a starting point, we define two decision problems for argument-incomplete argumentation frameworks.

s-Arg-Inc-Possible-Verification (**s**-ArgIncPV)	
Given:	An argument-incomplete argumentation framework $IAF = \langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ and a set $S \subseteq \mathscr{A}$.
Question:	Is S a possibly **s** extension of IAF?

s-Arg-Inc-Necessary-Verification (**s**-ArgIncNV)	
Given:	An argument-incomplete argumentation framework $IAF = \langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ and a set $S \subseteq \mathscr{A}$.
Question:	Is S a necessarily **s** extension of IAF?

Note that we do not make any restrictions on the choice of the set S in the problem instance, but restrict it to the arguments that occur in the completion when asking whether it is an extension. This captures the setting where all elements in $S \cap \mathcal{A}'$ must be contained in our final restriction of S, whereas this is not decided yet for the elements in $S \cap (\mathscr{A} \setminus \mathscr{A}')$. Furthermore, it is not a restriction that only elements from $\mathcal{A}'$ are sure to be in our final set, since other elements that should definitely be in the final set may be added to our argumentation framework in advance. Hence, this is a reasonable choice in argument-incomplete argumentation frameworks.

We will start the discussion of argument-incomplete argumentation frameworks with an easy result for conflict-freeness.

Proposition 1. cf-ArgIncPV *and* cf-ArgIncNV *both are in P.*

Proof. First, it holds that $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in$ cf-ArgIncPV if and only if $S|_{\mathscr{A}'}$ is conflict-free in $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$, because any conflict in $\langle \mathscr{A}', \mathscr{R}|_{\mathscr{A}'} \rangle$ can also be found in any other completion. Second, it holds that $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in$ cf-ArgIncNV if and only if S is conflict-free in $\langle \mathscr{A}, \mathscr{R} \rangle$, because if there is no conflict in $\langle \mathscr{A}, \mathscr{R} \rangle$, there is no other completion that can possibly have conflicts. $\square$

Now, we will turn to the other semantics. By definition of the decision problems, it is obvious that for $\mathbf{s} \in \{ad, st, cp, gr\}$, **s**-ArgIncPV is in NP and **s**-ArgIncNV is in coNP, as those four properties are easy to check. However, no polynomial-time algorithm is known to check for preferredness.

A trivial upper bound for PR-ArgIncPV can be obtained by the following—perhaps somewhat naive—approach: Check all supersets of the given set S as to whether they are admissible, and output "yes" if and only if the answer is always "no." Each of these checks is possible in polynomial time, and hence PR-ArgIncPV is in Σ_2^p. However, the problem PR-AttIncNV also is in coNP. As in the corresponding problem PR-ArgIncNV for attack-incomplete argumentation frameworks, the complement of PR-ArgIncNV is in NP. It is possible to check in polynomial time whether, given a completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*}\rangle$ of IAF and a set $S^* \subseteq \mathscr{A}^* : S|_{\mathcal{A}^*} \subset S^*$, either S^* is admissible or S is not admissible. All these trivial upper bounds are summarized in the following lemma.

Lemma 1. *1.* pr-ArgIncPV *is in* Σ_2^p.
2. For $\mathbf{s} \in \{$ ad, st, cp, gr$\}$, $\mathbf{s}$-ArgIncPV *is in NP.*
3. For $\mathbf{s} \in \{$ ad, st, cp, gr, pr$\}$, $\mathbf{s}$-ArgIncNV *is in coNP.*

We will now turn to showing lower bounds of these problems. We start with a straightforward reduction from PR-Verification to show coNP-hardness of the problems pr-ArgIncPV and pr-ArgIncNV.

Proposition 2. pr-ArgIncPV *is* coNP*-hard and the problem* pr-ArgIncNV *is* coNP*-complete.*

Proof. We show coNP-hardness by a reduction from the coNP-complete problem pr-Verification. Let $(\langle \mathscr{A}, \mathscr{R}\rangle, S)$ be a given instance of pr-Verification, and construct from it $(\langle \mathscr{A}, \mathscr{A}, \mathscr{R}\rangle, S)$, considered as an instance of both pr-ArgIncPV and pr-ArgIncNV. In the argument-incomplete argumentation framework, there are no arguments that can possibly join the discussion. Hence, the only completion in both cases is the argumentation framework $\langle \mathscr{A}, \mathscr{R}\rangle$. Now, it is easy to see that

$$\begin{aligned}&(\langle \mathscr{A}, \mathscr{R}\rangle, S) \in \text{pr-Verification}\\ \iff &(\langle \mathscr{A}, \mathscr{A}, \mathscr{R}\rangle, S) \in \text{pr-ArgIncPV}\\ \iff &(\langle \mathscr{A}, \mathscr{A}, \mathscr{R}\rangle, S) \in \text{pr-ArgIncNV}.\end{aligned}$$

This completes the proof. □

Theorem 1. ad-ArgIncPV *is* NP*-complete.*

Proof. As already mentioned, we only need to show NP-hardness. To this end, we reduce from the following NP-complete problem (see the book by Garey and Johnson [22]):

Exact-Cover-By-3-Sets (X3C)	
Given:	A set $B = \{b_1, \ldots, b_{3k}\}$ and a family $\mathscr{S}$ of subsets of B, with $\|S_j\| = 3$ for all $S_j \in \mathscr{S}$.
Question:	Does there exist a subfamily $\mathcal{S}' \subseteq \mathscr{S}$ of size k that exactly covers B, i.e., $\bigcup_{S_j \in \mathcal{S}'} S_j = B$?

Given an instance $(B, \mathscr{S}) = (\{b_1, \ldots, b_{3k}\}, \{S_1, \ldots, S_m\})$ of X3C, we construct an instance $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R}\rangle, S)$ of AD-ARGINCPV as follows:[2]

$$\begin{aligned}
\mathscr{A}' &= \{x\} \cup B, \\
\mathscr{A} &= \{x\} \cup B \cup \mathscr{S}, \\
\mathscr{R} &= \{(b_i, x) \mid b_i \in B\} \cup \\
&\quad \{(S_j, b_{j_1}), (S_j, b_{j_2}), (S_j, b_{j_3}) \mid S_j = \{b_{j_1}, b_{j_2}, b_{j_3}\} \in \mathscr{S}\} \cup \\
&\quad \{(S_i, S_j), (S_j, S_i) \mid S_i, S_j \in \mathscr{S} \text{ and } S_i \cap S_j \neq \emptyset\}, \\
S &= \{x\} \cup \mathscr{S}.
\end{aligned}$$

In particular, $\mathscr{A}$ contains one argument b_i for every element $b_i \in B$, $1 \leq i \leq 3k$, one argument S_j for every set S_j in $\mathscr{S}$, $1 \leq j \leq m$, and one additional argument x. All arguments corresponding to elements of B attack x, and each argument S_j attacks the three arguments corresponding to those elements of B that belong to S_j in $\mathscr{S}$. Additionally, there are attacks between S_i and S_j if the corresponding sets in $\mathscr{S}$ are not disjoint. Finally, $\mathscr{A}'$ and S act as opponents: x belongs to both, but the arguments corresponding to elements in B belong to $\mathscr{A}'$ only, whereas the arguments corresponding to the sets in $\mathscr{S}$ belong to S only. See Fig. 4 for two examples of this construction: Fig. 4a shows a yes-instance of AD-ARGINCPV created from a yes-instance of X3C, and Fig. 4b shows a no-instance of AD-ARGINCPV created from a no-instance of X3C.

We claim that $(B, \mathscr{S}) \in$ X3C if and only if $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R}\rangle, S) \in$ AD-ARGINCPV.

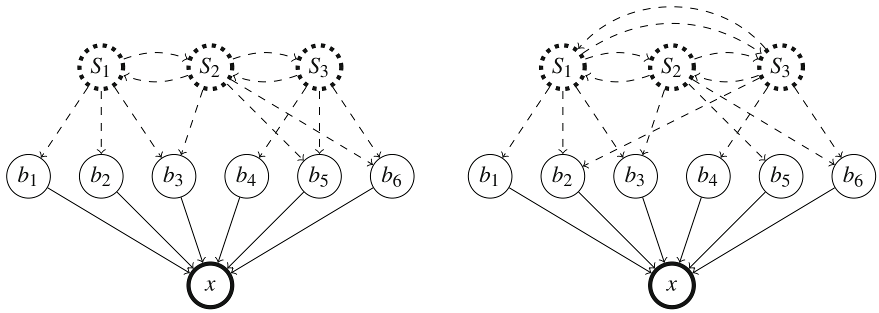
($\Longrightarrow$) Clearly, if $(B, \mathscr{S})$ is a yes-instance of X3C, we can add exactly those arguments S_i to $\mathscr{A}'$ that correspond to an exact cover of B. Let $\mathscr{A}^*$ be the argument set of this completion. In $\mathscr{A}^*$, every b_i, $1 \leq i \leq 3k$, is attacked by exactly one argument S_j, $1 \leq j \leq m$, as of the exact cover. Hence, $x \in S|_{\mathscr{A}^*}$ is defended against every attack. Additionally, the arguments S_j in $\mathscr{A}^*$ have no attacks between them, because the corresponding sets are pairwise disjoint, which implies that no new attacks on the elements of $S|_{\mathscr{A}^*}$ are introduced. But this means that $S|_{\mathscr{A}^*}$ is admissible in $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*}\rangle$.

($\Longleftarrow$) If there is a completion with the argument set $\mathscr{A}^*$, this completion must defend x against every b_i, $1 \leq i \leq 3k$. This means that there must exist a cover of the elements of B by the sets of $\mathscr{S}$. But because the arguments S_j attack each other whenever they are not disjoint, this cover must be exact; otherwise, the set $S|_{\mathscr{A}^*}$ would not be conflict-free. Hence, there exists an exact cover of B. □

We now try to tighten the bounds of **s**-ARGINCPV for each **s** ∈ {ST, CP, GR, PR}. The first step is proving NP-hardness in all four cases. By Lemma 1, this gives NP-completeness for **s** ∈ {ST, CP, GR}. Later on, we will use this results for **s** = PR, as well as the result for coNP-hardness, to show DP-hardness.

Theorem 2. *For* **s**∈ {ST, CP, GR}, **s**-ARGINCPV *is* NP-*complete, and* PR-ARGINCPV *is* NP-*hard.*

[2] We slightly abuse notation and use the same identifiers for both instances; it will always be clear from the context, though, which instance an element belongs to.



(a) $\mathscr{S} = \{\{b_1, b_2, b_3\}, \{b_3, b_5, b_6\}, \{b_4, b_5, b_6\}\}$. $(B, \mathscr{S})$ is a yes-instance of X3C that yields a yes-instance of AD-ARGINCPV.

(b) $\mathscr{S} = \{\{b_1, b_2, b_3\}, \{b_3, b_5, b_6\}, \{b_2, b_4, b_6\}\}$. $(B, \mathscr{S})$ is a no-instance of X3C that yields a no-instance of AD-ARGINCPV.

Fig. 4. Two examples of the reduction from X3C to AD-ARGINCPV. Both X3C instances have $B = \{b_1, \ldots, b_6\}$. All the arguments belong to $\mathscr{A}$, $\mathscr{A}'$ contains the solid arguments only, and the thick arguments are part of S.

Proof. Again, membership of the three former problems in NP is clear. It remains to show hardness for all four problems. We do this by showing that the reduction used in Theorem 1 also works for those four problems. To this end, we will prove that

$$
\begin{aligned}
&(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in \text{AD-ARGINCPV} \\
\iff &(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in \text{ST-ARGINCPV} \\
\iff &(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in \text{PR-ARGINCPV} \\
\iff &(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in \text{GR-ARGINCPV} \\
\iff &(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in \text{CP-ARGINCPV}
\end{aligned}
$$

holds for the instance $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S)$ constructed in the proof of Theorem 1.

($(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in$ AD-ARGINCPV implies $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in$ ST-ARGINCPV): If $S|_{\mathcal{A}^*}$ is admissible for a completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$, it in particular is conflict-free. We know from the reduction that $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ only contains arguments S_j that do not attack each other, and all these arguments belong to $S|_{\mathcal{A}^*}$. Hence, the only arguments outside of $S|_{\mathcal{A}^*}$ are the b_i's. But all of them are attacked, as explained in the proof of Theorem 1. Therefore, $S|_{\mathcal{A}^*}$ is a stable extension of $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$.

($(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in$ PR-ARGINCPV implies $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S) \in$ GR-ARGINCPV): If $S|_{\mathcal{A}^*}$ is preferred for a completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$, it is admissible, and thus the only arguments that are not attacked by any other argument are those S_j that correspond to an exact cover. This means for the characteristic function of this completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ that the output of the first step is the set that contains exactly those S_j. In the second step, we add argument x, because all those S_j defend x against all attacks from the arguments b_i. No new arguments are added in step three.

Therefore, this set is the grounded extension of the argumentation framework $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*}\rangle$. But this set is exactly the set $S|_{\mathcal{A}^*}$. Hence, $S|_{\mathcal{A}^*}$ is the grounded extension of $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*}\rangle$.

It is easy to see the three remaining implications needed to prove these five statements equivalent: Every stable set is preferred, every grounded set is complete, and every complete set is admissible. This completes the proof. □

We now strengthen the NP-hardness lower bound for PR-ARGINCPV given in Theorem 2 to DP-hardness. The following lemma due to Wagner [32] gives a sufficient condition for proving hardness for DP.

Lemma 2 (Wagner [32]). *Let A be some* NP*-hard problem, and let B be any set. If there exists a polynomial-time computable function f such that, for any two instances z_1 and z_2 of A for which $z_2 \in A$ implies $z_1 \in A$, we have*

$$(z_1 \in A \text{ and } z_2 \notin A) \iff f(z_1, z_2) \in B,$$

then B is DP*-hard.*

Theorem 3. PR-ARGINCPV *is* DP*-hard.*

Proof. We will use Wagner's lemma to show DP-hardness: Let PR-ARGINCPV be the set B from Wagner's lemma, and let X3C be the NP-complete problem A in that lemma. Let z_1 and z_2 be two instances of X3C such that $z_2 \in$ X3C implies $z_1 \in$ X3C. We construct an instance $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R}\rangle, S)$ of PR-ARGINCPV as follows:

- Construct an instance $(\langle \mathscr{A}'_1, \mathscr{A}_1, \mathscr{R}_1\rangle, S_1)$ from the X3C instance z_1 exactly as in the proof of Theorem 1.
- The construction of an instance $(\langle \mathscr{A}'_2, \mathscr{A}_2, \mathscr{R}_2\rangle, S_2)$ from the X3C instance z_2, however, is obtained as the composition of two reductions: Since PR-VERIFICATION is coNP-complete and X3C is NP-complete, there exists a reduction f such that $z_2 \notin$ X3C if and only if $f(z_2) \in$ PR-VERIFICATION. Now, letting g be the reduction from Proposition 2, we have $z_2 \notin$ X3C if and only if $g(f(z_2)) \in$ PR-ARGINCPV.
- Given two instances of PR-ARGINCPV, $(\langle \mathscr{A}'_1, \mathscr{A}_1, \mathscr{R}_1\rangle, S_1)$ and $(\langle \mathscr{A}'_2, \mathscr{A}_2, \mathscr{R}_2\rangle, S_2)$, let $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R}\rangle, S) = (\langle \mathscr{A}'_1 \cup \mathscr{A}'_2, \mathscr{A}_1 \cup \mathscr{A}_2, \mathscr{R}_1 \cup \mathscr{R}_2\rangle, S_1 \cup S_2)$ if $\mathscr{A}_1 \cap \mathscr{A}_2 = \emptyset$ (otherwise, simply rename the elements in one instance). Hence, this new instance consists of two disconnected argument-incomplete argumentation frameworks.

This completes the reduction. We claim that ($z_1 \in$ X3C and $z_2 \notin$ X3C) if and only if $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R}\rangle, S) \in$ PR-ARGINCPV.

($\Longrightarrow$) If $z_1 \in$ X3C and $z_2 \notin$ X3C, then $(\langle \mathscr{A}'_1, \mathscr{A}_1, \mathscr{R}_1\rangle, S_1)$ and $(\langle \mathscr{A}'_2, \mathscr{A}_2, \mathscr{R}_2\rangle, S_2)$ both are yes-instances of PR-ARGINCPV. Thus we must have a completion for the first and a completion for the second argument-incomplete argumentation framework such that S_1 restricted to the arguments in this first completion and S_2 restricted to the arguments in the second completion are preferred in their respective completion. But then, using the same completions for each part of $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$, we have that S restricted to those arguments must be preferred in this argumentation framework. This is true because no new attacks are introduced in $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ and, therefore, neither are any new conflicts added nor do the elements of S have to be defended by any other arguments than before. Hence, $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S)$ is a yes-instance of PR-ARGINCPV.

($\Longleftarrow$) Conversely, assume that $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S)$ is a yes-instance of PR-ARGINCPV, and assume further that $(\langle \mathscr{A}_i', \mathscr{A}_i, \mathscr{R}_i \rangle, S_i)$ is a no-instance of PR-ARGINCPV for some $i \in \{1, 2\}$. Then there is no completion $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$ of $\langle \mathscr{A}_i', \mathscr{A}_i, \mathscr{R}_i \rangle$ such that $S_i|_{\mathscr{A}_i^*}$ is preferred in it. That means that for every completion $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$, $S_i|_{\mathscr{A}_i^*}$ either is not conflict-free, or is not admissible, or that there exists a superset of $S_i|_{\mathscr{A}_i^*}$ in $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$ that is admissible. We consider these cases separately:

1. If $S_i|_{\mathscr{A}_i^*}$ is not conflict-free in $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$, this conflict also exists in $S|_{\mathscr{A}^*}$ for any completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ of $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ with $\mathscr{A}^* \cap \mathscr{A}_i = \mathscr{A}_i^*$.
2. If $S_i|_{\mathscr{A}_i^*}$ is not admissible in $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$, there must be an undefended attack. However, by the same argument as above, this attack is still undefended in any completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ of $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ with $\mathscr{A}^* \cap \mathscr{A}_i = \mathscr{A}_i^*$.
3. If there is a superset of $S_i|_{\mathscr{A}_i^*}$ preventing it from being preferred in $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$, this superset translates into a superset of $S|_{\mathscr{A}^*}$ for any completion $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$ of $\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle$ with $\mathscr{A}^* \cap \mathscr{A}_i = \mathscr{A}_i^*$, thus also preventing $S|_{\mathscr{A}^*}$ from being preferred in $\langle \mathscr{A}^*, \mathscr{R}|_{\mathscr{A}^*} \rangle$.

Hence, none of these cases can happen, because $(\langle \mathscr{A}', \mathscr{A}, \mathscr{R} \rangle, S)$ is a yes-instance of PR-ARGINCPV. But this means that $S_i|_{\mathscr{A}_i^*}$ is a preferred extension of a completion $\langle \mathscr{A}_i^*, \mathscr{R}_i|_{\mathscr{A}_i^*} \rangle$ of $\langle \mathscr{A}_i', \mathscr{A}_i, \mathscr{R}_i \rangle$, a contradiction. □

4 Conclusions and Open Questions

We have analyzed a setting for argumentation frameworks in which only a subset of all arguments is currently known to be part of the discussion. To this end, we have introduced a formal model for argument-incomplete argumentation frameworks including adaptions of the criteria of conflict-freeness, admissibility, preferredness, stability, completeness, and groundedness. These adaptions were defined by means of the notions of possibility and necessity. On this basis, we adapted the decision problem **s**-VERIFICATION and defined two variants, namely **s**-ARGINCPV and **s**-sARGINCNV, that fit our model.

Table 1 summarizes already known results for the **s**-VERIFICATION problem due to Dung [18], Dunne and Wooldridge [19], and Dimopoulos and Torres [16], as well as our results. In contrast to the results of **s**-VERIFICATION, **s**-ARGINCPV is hard to decide in all cases, except for the trivial property conflict-freeness. Besides the straightforward results for conflict-freeness and preferredness, the exact complexity of **s**-ARGINCNV remains open, as well as that of PR-ARGINCPV.

As a future task, we propose to investigate other decision problems, e.g., CREDULOUS-ACCEPTANCE, SKEPTICAL-ACCEPTANCE, EXISTENCE, and NON-EMPTINESS, adapt their notion to fit our model, and analyze their complexity.

Table 1. Overview of complexity results both in the standard model (**s**-Verification) and in the argument-incomplete model of this paper (**s**-ArgIncPV and **s**-ArgIncNV)

s	Verification	ArgIncPV	ArgIncNV
CF	in P	in P (Lemma 1)	in P (Lemma 1)
AD	in P	NP-complete (Theorem 1)	in coNP (Lemma 1)
ST	in P	NP-complete (Theorem 2)	in coNP (Lemma 1)
CP	in P	NP-complete (Theorem 2)	in coNP (Lemma 1)
GR	in P	NP-complete (Theorem 2)	in coNP (Lemma 1)
PR	coNP-complete	DP-hard (Theorem 3), in Σ_2^p (Lemma 1)	coNP-complete (Lemma 2)

On the other hand, it would be interesting to generalize other semantics (e.g., the ideal, semi-stable, or prudent semantics [19]) in the context of argument-incomplete argumentation frameworks.

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